Bottom up parsing

- Construct a parse tree for an input string beginning at leaves and going towards root OR
- Reduce a string w of input to start symbol of grammar

Consider a grammar

$S \rightarrow aABe$

$A \rightarrow Abc \;|\; b$

$B \rightarrow d$

And reduction of a string

$a\ \underline{b}\ b\ c\ d\ e$

$a\ \underline{A\ b\ c}\ d\ e$

$a\ A\ \underline{d}\ e$

$\underline{a\ A\ B\ e}$

S

The sentential forms happen to be a *right most derivation in the reverse order.*

$S \Rightarrow a\ A\ \underline{B}\ e$

$\Rightarrow a\ \underline{A}\ d\ e$

$\Rightarrow a\ \underline{A}\ b\ c\ d\ e$

$\Rightarrow a\ b\ b\ c\ d\ e$

Shift reduce parsing

- Split string being parsed into two parts
  - Two parts are separated by a special character "."
  - Left part is a string of terminals and non terminals
  - Right part is a string of terminals

- Initially the input is .w

Shift reduce parsing …

- Bottom up parsing has two actions
- Shift: move terminal symbol from right string to left string

  if string before shift is $\alpha.pqr$

  then string after shift is $\alpha p.qr$

Shift reduce parsing …

- Reduce: immediately on the left of "." identify a string same as RHS of a production and replace it by LHS

if string before reduce action is $\alpha\beta.pqr$

and $A \rightarrow \beta$ is a production

then string after reduction is $\alpha A.pqr$

Example

Assume grammar is E → E+E | E*E | id

Parse id*id+id

Assume an oracle tells you when to shift / when to reduce

String	**action (by oracle)**
.id*id+id	shift
id.*id+id	reduce E→id
E.*id+id	shift
E*.id+id	shift
E*id.+id	reduce E→id
E*E.+id	reduce E→E*E
E.+id	shift
E+.id	shift
E+id.	Reduce E→id
E+E.	Reduce E→E+E
E.	ACCEPT

Shift reduce parsing …

- Symbols on the left of "." are kept on a stack
  - Top of the stack is at "."
  - Shift pushes a terminal on the stack
  - Reduce pops symbols (rhs of production) and pushes a non terminal (lhs of production) onto the stack
- The most important issue: when to shift and when to reduce
- Reduce action should be taken only if the result can be reduced to the start symbol

Issues in bottom up parsing

- How do we know which action to take
  - whether to shift or reduce
  - Which production to use for reduction?
- Sometimes parser can reduce but it should not:

  $X \rightarrow \epsilon$ can always be used for reduction!

Issues in bottom up parsing

- Sometimes parser can reduce in different ways!
- Given stack δ and input symbol a, should the parser
  - Shift a onto stack (making it δa)
  - Reduce by some production $A \rightarrow \beta$ assuming that stack has form $\alpha\beta$ (making it αA)
  - Stack can have many combinations of $\alpha\beta$
  - How to keep track of length of β?

Handles

- The basic steps of a bottom-up parser are
  - to identify a *substring* within a *rightmost sentential* form which matches the RHS of a rule.
  - when this substring is replaced by the LHS of the matching rule, it must produce the previous rightmost-sentential form.
- Such a substring is called a *handle*

Handle

- A *handle* of a right sentential form γ is
  - a production rule $A \rightarrow \beta$, and
  - an occurrence of a sub-string β in γ

such that

- when the occurrence of β is replaced by A in γ, we get the previous right sentential form in a rightmost derivation of γ.

Handle

Formally, if

$$S \Rightarrow^{rm*} \alpha A w \Rightarrow^{rm} \alpha\beta w,$$

then

- β in the position following α,
- and the corresponding production $A \rightarrow \beta$

is a handle of $\alpha\beta w$.

- The string w consists of only terminal symbols

Handle

- We only want to reduce handle and not any RHS
- Handle pruning: If β is a handle and $A \rightarrow \beta$ is a production then replace β by A
- A right most derivation in reverse can be obtained by handle pruning.

Handle: Observation

- *Only terminal symbols can appear to the right of a handle in a rightmost sentential form.*
- Why?

Handle: Observation

Is this scenario possible:

- $\alpha\beta\gamma$ is the content of the stack
- $A \rightarrow \gamma$ is a handle
- The stack content reduces to $\alpha\beta A$
- Now $\mathrm{B} \rightarrow \beta$ is the handle

In other words, handle is not on top, but buried *inside* stack

Not Possible! Why?

Handles …

- Consider two cases of right most derivation to understand the fact that handle appears on the top of the stack

$$S \rightarrow \alpha A z \rightarrow \alpha \beta B y z \rightarrow \alpha \beta \gamma y z$$
$$S \rightarrow \alpha B x A z \rightarrow \alpha B x y z \rightarrow \alpha \gamma x y z$$

Handle always appears on the top

Case I: $S \to \alpha A z \to \alpha\beta B y z \to \alpha\beta\gamma y z$

stack	input	action
$\alpha\beta\gamma$	yz	reduce by B→γ
$\alpha\beta B$	yz	shift y
$\alpha\beta By$	z	reduce by A→ βBy
αA	z	

Case II: $S \to \alpha B x A z \to \alpha B x y z \to \alpha\gamma x y z$

stack	input	action
$\alpha\gamma$	xyz	reduce by B→γ
αB	xyz	shift x
αBx	yz	shift y
αBxy	z	reduce A→y
αBxA	z	

Shift Reduce Parsers

- The general shift-reduce technique is:
  - if there is no handle on the stack then shift
  - If there is a handle then reduce
- Bottom up parsing is essentially the process of detecting handles and reducing them.
- Different bottom-up parsers differ in the way they detect handles.

Conflicts

- What happens when there is a choice
  - What action to take in case both shift and reduce are valid?

    shift-reduce conflict
  - Which rule to use for reduction if reduction is possible by more than one rule?

    reduce-reduce conflict

Conflicts

- Conflicts come either because of ambiguous grammars or parsing method is not powerful enough

Shift reduce conflict

Consider the grammar $E \rightarrow E+E \mid E*E \mid id$

and the input id+id*id

stack	input	action
E+E	*id	reduce by E→E+E
E	*id	shift
E*	id	shift
E*id		reduce by E→id
E*E		reduce byE→E*E
E		

stack	input	action
E+E	*id	shift
E+E*	id	shift
E+E*id		reduce by E→id
E+E*E		reduce by E→E*E
E+E		reduce by E→E+E
E		

Reduce reduce conflict

Consider the grammar $M \rightarrow R+R \mid R+c \mid R$

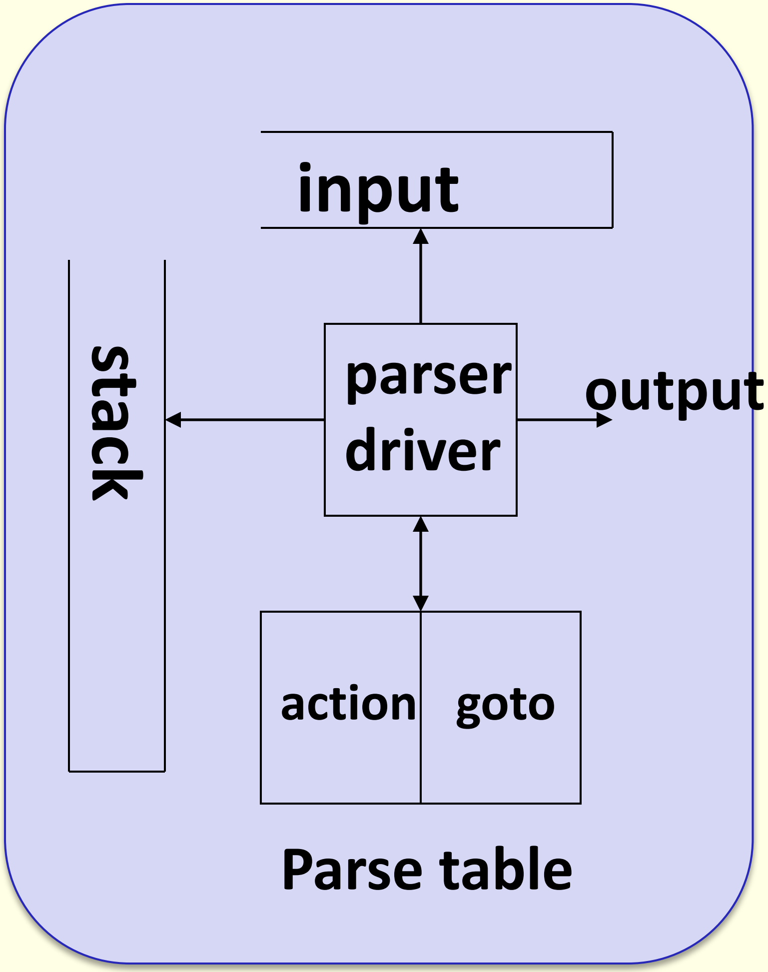
$R \rightarrow c$

and the input $c+c$

Stack	input	action
	c+c	shift
c	+c	reduce by R→c
R	+c	shift
R+	c	shift
R+c		reduce by R→c
R+R		reduce by M→R+R
M		

Stack	input	action
	c+c	shift
c	+c	reduce by R→c
R	+c	shift
R+	c	shift
R+c		reduce by M→R+c
M		

LR parsing



- Input buffer contains the input string.
- Stack contains a string of the form $S_0X_1S_1X_2......X_nS_n$ where each X_i is a grammar symbol and each S_i is a state.
- Table contains action and goto parts.
- action table is indexed by state and terminal symbols.
- goto table is indexed by state and non terminal symbols.

Example

Consider a grammar and its parse table

E → E + T | T
T → T * F | F
F → (E) | id

State	id	+	*	(	)	$	E	T	F
0	s5			s4			1	2	3
1		s6				acc			
2		r2	s7		r2	r2			
3		r4	r4		r4	r4			
4	s5			s4			8	2	3
5		r6	r6		r6	r6			
6	s5			s4				9	3
7	s5			s4					10
8		s6			s11				
9		r1	s7		r1	r1			
10		r3	r3		r3	r3			
11		r5	r5		r5	r5			

action

goto

Actions in an LR (shift reduce) parser

- Assume S_i is top of stack and a_i is current input symbol

- Action $[S_i, a_i]$ can have four values
  1. sj: shift a_i to the stack, goto state S_j
  2. rk: reduce by rule number k
  3. acc: Accept
  4. err: Error (empty cells in the table)

Driving the LR parser

Stack: $S_0X_1S_1X_2 \ldots X_mS_m$ Input: $a_ia_{i+1} \ldots a_n\$$

- If $action[S_m,a_i]$ = shift S
  Then the configuration becomes
  Stack: $S_0X_1S_1 \ldots \ldots X_mS_ma_iS$ Input: $a_{i+1} \ldots a_n\$$

- If $action[S_m,a_i]$ = reduce $A \rightarrow \beta$
  Then the configuration becomes
  Stack: $S_0X_1S_1 \ldots X_{m-r}S_{m-r}AS$ Input: $a_ia_{i+1} \ldots a_n\$$
  Where $r = |\beta|$ and $S = goto[S_{m-r},A]$

Driving the LR parser

Stack: $S_0X_1S_1X_2\ldots X_mS_m$ Input: $a_ia_{i+1}\ldots a_n\$$

- If action$[S_m,a_i]$ = accept
  Then parsing is completed. HALT

- If action$[S_m,a_i]$ = error (or empty cell)
  Then invoke error recovery routine.

Parse id + id * id

Stack	Input	Action
0	id+id*id$	shift 5
0 id 5	+id*id$	reduce by F→id
0 F 3	+id*id$	reduce by T→F
0 T 2	+id*id$	reduce by E→T
0 E 1	+id*id$	shift 6
0 E 1 + 6	id*id$	shift 5
0 E 1 + 6 id 5	*id$	reduce by F→id
0 E 1 + 6 F 3	*id$	reduce by T→F
0 E 1 + 6 T 9	*id$	shift 7
0 E 1 + 6 T 9 * 7	id$	shift 5
0 E 1 + 6 T 9 * 7 id 5	$	reduce by F→id
0 E 1 + 6 T 9 * 7 F 10	$	reduce by T→T*F
0 E 1 + 6 T 9	$	reduce by E→E+T
0 E 1	$	ACCEPT

Configuration of a LR parser

- The tuple

  <Stack Contents, Remaining Input>

  defines a *configuration* of a LR parser
- Initially the configuration is

  $$<S_0 , \quad a_0a_1\ldots a_n\$ >$$

- Typical final configuration on a successful parse is

  $$< S_0X_1S_i , \quad \$>$$

LR parsing Algorithm

Initial state: Stack: S_0 Input: w$

```
while (1) {
    if (action[S,a] = shift S') {
        push(a); push(S'); ip++
    } else if (action[S,a] = reduce A→β) {
        pop (2*|β|) symbols;
        push(A); push (goto[S'',A])
        (S'' is the state at stack top after popping symbols)
    } else if (action[S,a] = accept) {
        exit
    } else { error }
```

Constructing parse table

Augment the grammar

- G is a grammar with start symbol S
- The augmented grammar G' for G has a new start symbol S' and an additional production S' → S
- When the parser reduces by this rule it will stop with accept

Production to Use for Reduction

- How do we know which production to apply in a given configuration
- We can guess!
  - May require backtracking
- Keep track of “ALL” possible rules that can apply at a given point in the input string
  - But in general, there is no upper bound on the length of the input string
  - Is there a bound on number of applicable rules?

Some hands on!

- $E' \rightarrow E$
- $E \rightarrow E + T$
- $E \rightarrow T$
- $T \rightarrow T * F$
- $T \rightarrow F$
- $F \rightarrow (E)$
- $F \rightarrow id$

Strings to Parse

- id + id + id + id
- id * id * id * id
- id * id + id * id
- id * (id + id) * id

Parser states

- Goal is to know the valid reductions at any given point
- Summarize all possible stack prefixes α as a parser state
- Parser state is defined by a DFA state that reads in the stack α
- Accept states of DFA are unique reductions

Viable prefixes

- α is a viable prefix of the grammar if
  - $\exists w$ such that αw is a right sentential form
  - $\langle \alpha, w \rangle$ is a configuration of the parser
- As long as the parser has viable prefixes on the stack no parser error has been seen
- The set of viable prefixes is a regular language
- We can construct an automaton that accepts viable prefixes

LR(0) items

- An LR(0) item of a grammar G is a production of G with a special symbol "." at some position of the right side
- Thus production A→XYZ gives four LR(0) items

  A → .XYZ

  A → X.YZ

  A → XY.Z

  A → XYZ.

LR(0) items

- An item indicates how much of a production has been seen at a point in the process of parsing
  - Symbols on the left of "." are already on the stacks
  - Symbols on the right of "." are expected in the input

Start state

- Start state of DFA is an empty stack corresponding to $S' \rightarrow .S$ item
- This means no input has been seen
- The parser expects to see a string derived from S

Closure of a state

- **Closure** of a state adds items for all productions whose LHS occurs in an item in the state, just after "."
  - Set of possible productions to be reduced next
  - Added items have "." located at the beginning
  - No symbol of these items is on the stack as yet

Closure operation

- Let I be a set of items for a grammar G
- closure(I) is a set constructed as follows:
  - Every item in I is in closure (I)
  - If $A \rightarrow \alpha.B\beta$ is in closure(I) and $B \rightarrow \gamma$ is a production then $B \rightarrow .\gamma$ is in closure(I)
- Intuitively $A \rightarrow \alpha.B\beta$ indicates that we expect a string derivable from $B\beta$ in input
- If $B \rightarrow \gamma$ is a production then we might see a string derivable from γ at this point

Example

For the grammar

$E' \rightarrow E$

$E \rightarrow E + T \mid T$

$T \rightarrow T * F \mid F$

$F \rightarrow (E) \mid id$

If I is { $E' \rightarrow .E$ } then closure(I) is

$E' \rightarrow .E$

$E \rightarrow .E + T$

$E \rightarrow .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .id$

$F \rightarrow .(E)$

Goto operation

- Goto(I,X) , where I is a set of items and X is a grammar symbol,
  - is closure of set of item $A \rightarrow \alpha X.\beta$
  - such that $A \rightarrow \alpha.X\beta$ is in I
- Intuitively if I is a set of items for some valid prefix α then goto(I,X) is set of valid items for prefix αX

Goto operation

If I is { $E' \rightarrow E.$, $E \rightarrow E. + T$ } then goto(I,+) is

$E \rightarrow E + .T$

$T \rightarrow .T * F$

$T \rightarrow .F$

$F \rightarrow .(E)$

$F \rightarrow .id$

Sets of items

C : Collection of sets of LR(0) items for grammar G'

C = { closure ({ S' $\rightarrow$ .S }) }

repeat

  for each set of items I in C

    for each grammar symbol X

      if goto (I,X) is not empty and not in C

        ADD goto(I,X) to C

until no more additions to C

Example

Grammar:

$E' \rightarrow E$
$E \rightarrow E+T \mid T$
$T \rightarrow T*F \mid F$
$F \rightarrow (E) \mid id$

I_0: closure($E' \rightarrow .E$)

$E' \rightarrow .E$
$E \rightarrow .E + T$
$E \rightarrow .T$
$T \rightarrow .T * F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_1: goto(I_0,E)

$E' \rightarrow E.$
$E \rightarrow E. + T$

I_2: goto(I_0,T)

$E \rightarrow T.$
$T \rightarrow T. *F$

I_3: goto(I_0,F)

$T \rightarrow F.$

I_4: goto(I_0,( )

$F \rightarrow (.E)$
$E \rightarrow .E + T$
$E \rightarrow .T$
$T \rightarrow .T * F$
$T \rightarrow .F$
$F \rightarrow .(E)$
$F \rightarrow .id$

I_5: goto(I_0,id)

$F \rightarrow id.$

I_6: goto(I_1,+)

E → E + .T

T → .T * F

T → .F

F → .(E)

F → .id

I_7: goto(I_2,*)

T → T * .F

F →.(E)

F → .id

I_8: goto(I_4,E)

F → (E.)

E → E. + T

goto(I_4,T) is I_2
goto(I_4,F) is I_3
goto(I_4,( ) is I_4
goto(I_4,id) is I_5

I_9: goto(I_6,T)

E → E + T.

T → T. * F

goto(I_6,F) is I_3
goto(I_6,( ) is I_4
goto(I_6,id) is I_5

I_{10}: goto(I_7,F)

T → T * F.

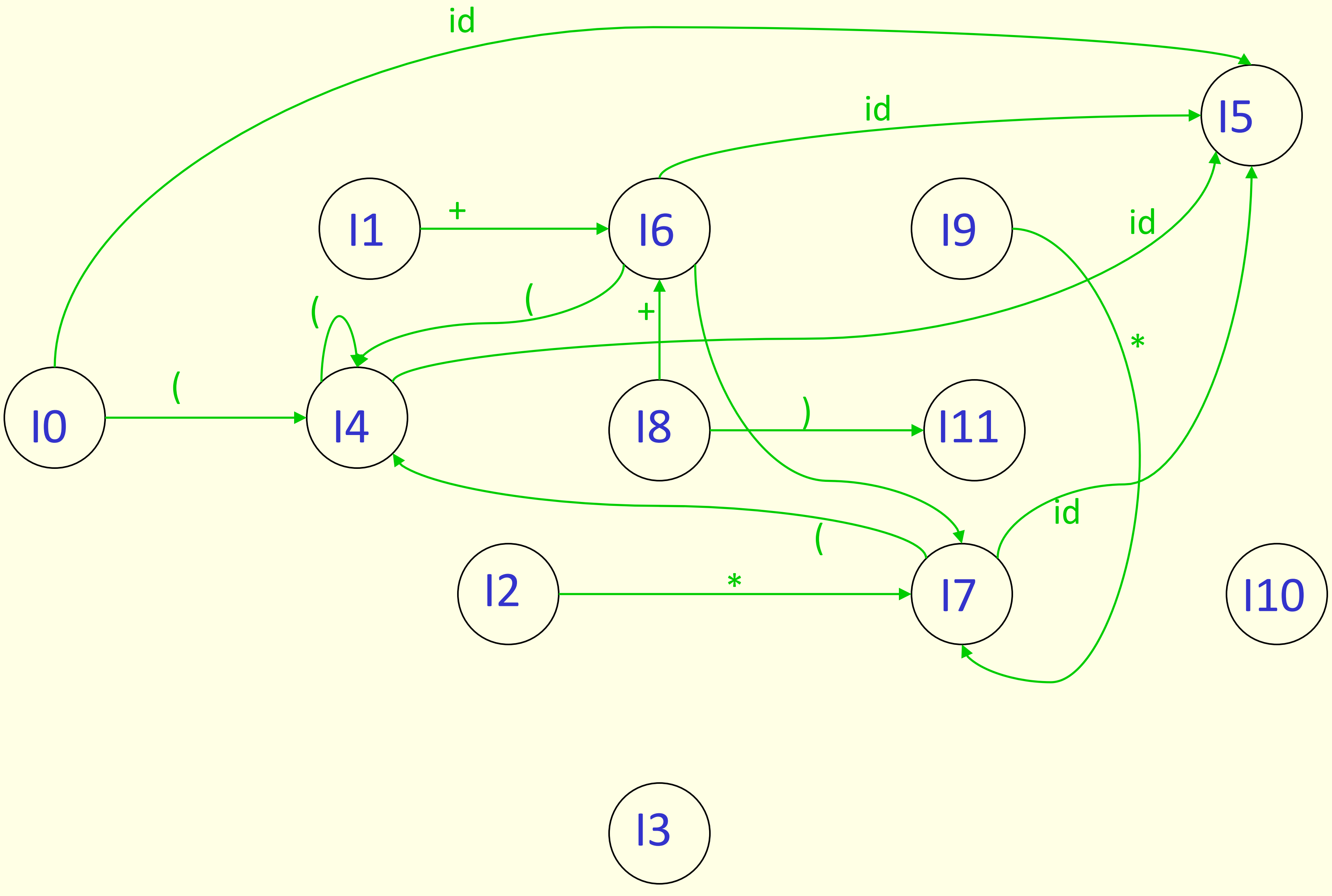
goto(I_7,( ) is I_4
goto(I_7,id) is I_5

I_{11}: goto(I_8,))

F → (E).

goto(I_8,+) is I_6

goto(I_9,*) is I_7



id
id
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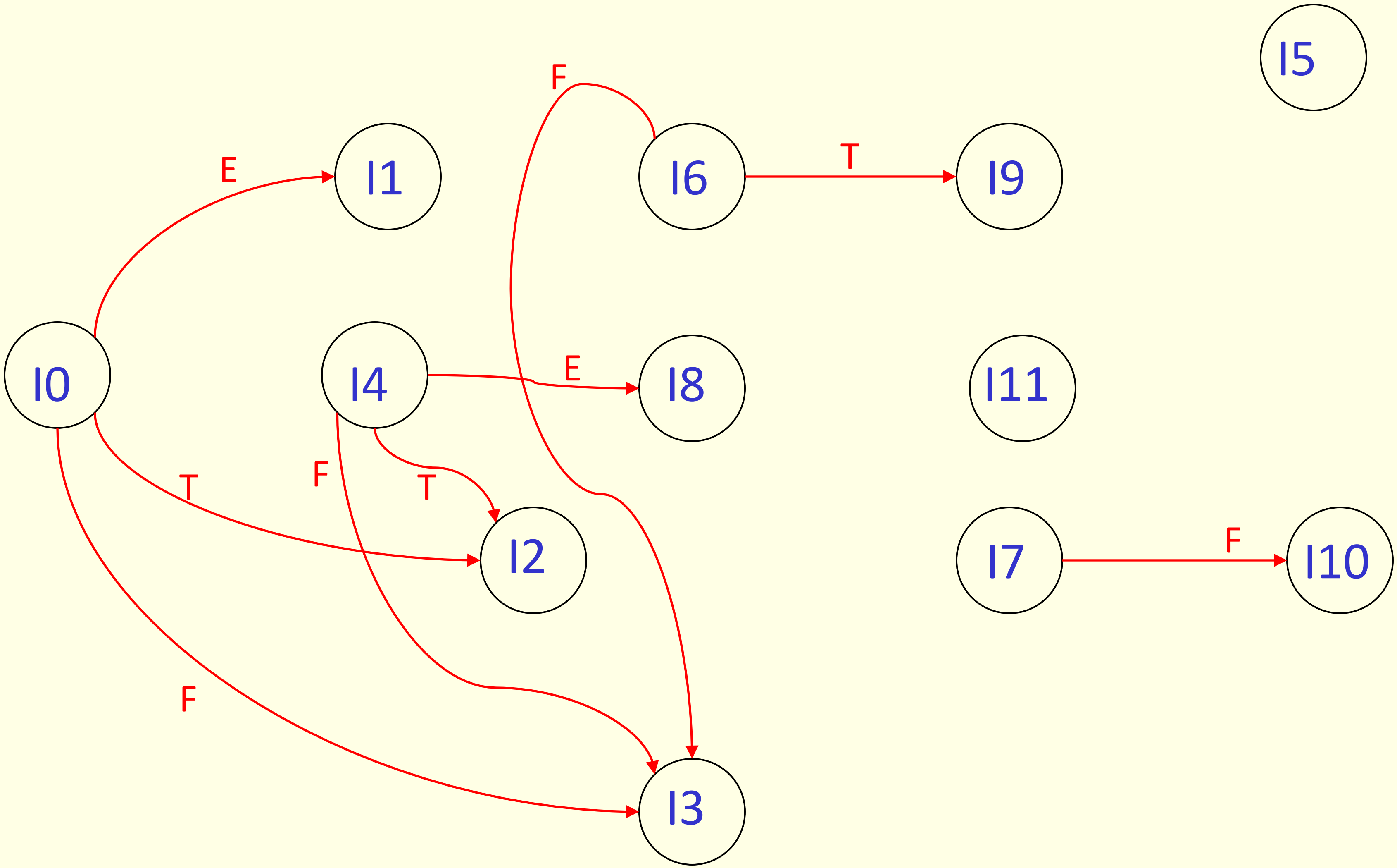
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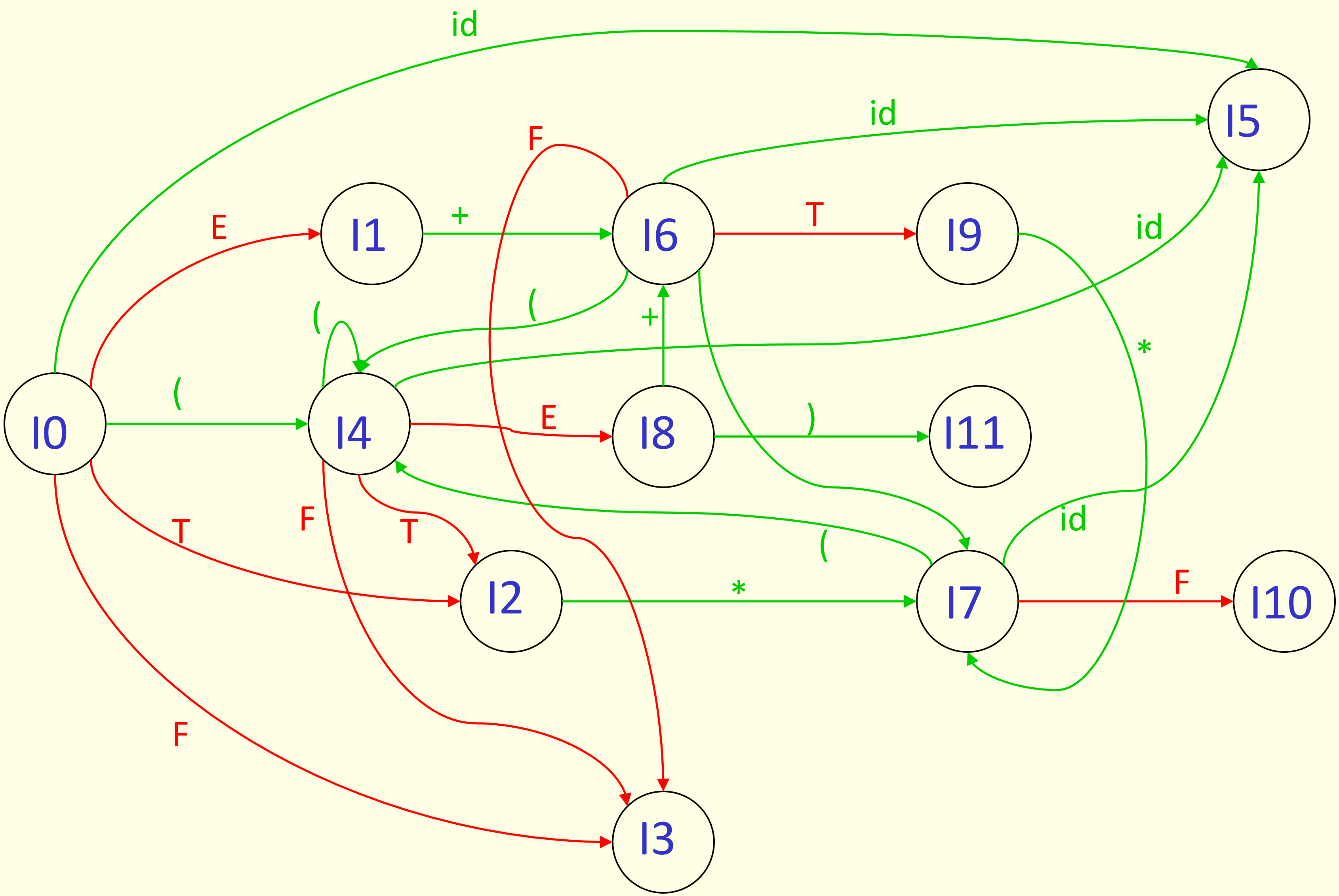
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LR(0) (?) Parse Table

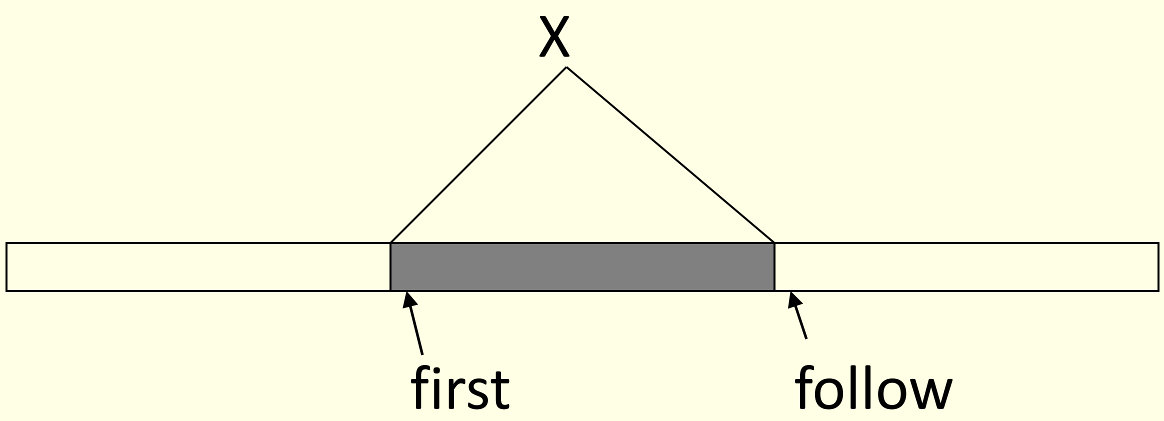
- The information is still not sufficient to help us resolve shift-reduce conflict. For example the state:

$$
\begin{aligned}
I_1: E' &\rightarrow E. \\
E &\rightarrow E. + T
\end{aligned}
$$

- We need some more information to make decisions.

Constructing parse table

- First(α) for a string of terminals and non terminals α is
  - Set of symbols that might begin the fully expanded (made of only tokens) version of α
- Follow(X) for a non terminal X is
  - set of symbols that might follow the derivation of X in the input stream



Compute first sets

- If X is a terminal symbol then first(X) = {X}
- If $X \rightarrow \epsilon$ is a production then ϵ is in first(X)
- If X is a non terminal and $X \rightarrow Y_l Y_2 \ldots Y_k$ is a production, then

  if for some i, a is in first(Y_i)

  and ϵ is in all of first(Y_j) (such that $j<i$)

  then a is in first(X)
- If ϵ is in first (Y_1) … first(Y_k) then ϵ is in first(X)
- Now generalize to a string α of terminals and non-terminals

Example

- For the expression grammar

  $E \rightarrow T\ E'$ $\qquad$ $E' \rightarrow +T\ E' \mid \epsilon$

  $T \rightarrow F\ T'$ $\qquad$ $T' \rightarrow *\ F\ T' \mid \epsilon$

  $F \rightarrow (\ E\ ) \mid id$

  First(E) = First(T) = First(F)

  = { (, id }

  First(E')

  = {+, Є}

  First(T')

  = { *, Є}

Compute follow sets

1. Place \$ in follow(S) // S is the start symbol
2. If there is a production $A \rightarrow \alpha B \beta$
   then everything in first(β) (except ε) is in follow(B)
3. If there is a production $A \rightarrow \alpha B \beta$ and first(β) contains ε
   then everything in follow(A) is in follow(B)
4. If there is a production $A \rightarrow \alpha B$
   then everything in follow(A) is in follow(B)

Last two steps have to be repeated until the follow sets converge.

Example

- For the expression grammar

  $E \rightarrow T\ E'$

  $E' \rightarrow +\ T\ E'\ |\ \epsilon$

  $T \rightarrow F\ T'$

  $T' \rightarrow *\ F\ T'\ |\ \epsilon$

  $F \rightarrow (\ E\ )\ |\ id$

  follow(E) = follow(E') = { \$,) }
  follow(T) = follow(T') = { \$,), + }
  follow(F) = { \$,), +, *}

Construct SLR parse table

- Construct $C=\{I_0, \ldots, I_n\}$ the collection of sets of LR(0) items
- If $A \rightarrow \alpha.a\beta$ is in I_i and $goto(I_i, a) = I_j$
  then action[i,a] = shift j
- If $A \rightarrow \alpha.$ is in I_i
  then action[i,a] = reduce $A \rightarrow \alpha$ for all a in follow(A)
- If $S' \rightarrow S.$ is in I_i then action[i,$] = accept
- If $goto(I_i, A) = I_j$
  then goto[i,A]=j for all non terminals A
- All entries not defined are errors

Notes

- This method of parsing is called SLR (Simple LR)
- LR parsers accept LR(k) languages
  - L stands for left to right scan of input
  - R stands for rightmost derivation
  - k stands for number of lookahead token
- SLR is the simplest of the LR parsing methods. SLR is too weak to handle most languages!
- If an SLR parse table for a grammar does not have multiple entries in any cell then the grammar is unambiguous
- All SLR grammars are unambiguous
- Are all unambiguous grammars in SLR?

Practice Assignment

Construct SLR parse table for following grammar

$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid (E) \mid digit$

Show steps in parsing of string

9*5+(2+3*7)

- Steps to be followed
  - Augment the grammar
  - Construct set of LR(0) items
  - Construct the parse table
  - Show states of parser as the given string is parsed

Example

- Consider following grammar and its SLR parse table:

  $S' \rightarrow S$

  $S \rightarrow L = R$

  $S \rightarrow R$

  $L \rightarrow *R$

  $L \rightarrow id$

  $R \rightarrow L$

I_0: $S' \rightarrow .S$

$S \rightarrow .L=R$

$S \rightarrow .R$

$L \rightarrow .*R$

$L \rightarrow .id$

$R \rightarrow .L$

I_1: goto(I_0, S)

$S' \rightarrow S.$

I_2: goto(I_0, L)

$S \rightarrow L.=R$

$R \rightarrow L.$

Assignment (not to be submitted): Construct rest of the items and the parse table.

SLR parse table for the grammar

	=	*	id	$	S	L	R
0		s4	s5		1	2	3
1				acc			
2	s6,r6			r6			
3				r3			
4		s4	s5			8	7
5	r5			r5			
6		s4	s5			8	9
7	r4			r4			
8	r6			r6			
9				r2			

The table has multiple entries in action[2,=]

- There is both a shift and a reduce entry in action[2,=]. Therefore state 2 has a shift-reduce conflict on symbol “=“, However, the grammar is not ambiguous.
- Parse id=id assuming reduce action is taken in [2,=]

Stack	**input**	**action**
0	id=id	shift 5
0 id 5	=id	reduce by L$\rightarrow$id
0 L 2	=id	reduce by R$\rightarrow$L
0 R 3	=id	error

- if shift action is taken in [2,=]

Stack	**input**	**action**
0	id=id$	shift 5
0 id 5	=id$	reduce by L→id
0 L 2	=id$	shift 6
0 L 2 = 6	id$	shift 5
0 L 2 = 6 id 5	$	reduce by L→id
0 L 2 = 6 L 8	$	reduce by R→L
0 L 2 = 6 R 9	$	reduce by S→L=R
0 S 1	$	ACCEPT

Problems in SLR parsing

- No sentential form of this grammar can start with R=…
- However, the reduce action in action[2,=] generates a sentential form starting with R=
- Therefore, the reduce action is incorrect
- In SLR parsing method state i calls for reduction on symbol "a", by rule $A \rightarrow \alpha$ if I_i contains $[A \rightarrow \alpha.]$ and "a" is in follow(A)
- However, when state I appears on the top of the stack, the viable prefix $\beta\alpha$ on the stack may be such that βA can not be followed by symbol "a" in any right sentential form
- Thus, the reduction by the rule $A \rightarrow \alpha$ on symbol "a" is invalid
- SLR parsers cannot remember the left context

Canonical LR Parsing

- Carry extra information in the state so that wrong reductions by $A \rightarrow \alpha$ will be ruled out
- Redefine LR items to include a terminal symbol as a second component (look ahead symbol)
- The general form of the item becomes $[A \rightarrow \alpha.\beta, a]$ which is called LR(1) item.
- Item $[A \rightarrow \alpha., a]$ calls for reduction only if next input is a. The set of symbols “a”s will be a subset of Follow(A).

Closure(I)

repeat
    for each item [A $\rightarrow$ α.Bβ, a] in I
        for each production B $\rightarrow$ γ in G'
        and for each terminal b in First(βa)
            add item [B $\rightarrow$.γ, b] to I
until no more additions to I

Example

Consider the following grammar

S' → S
S → CC
C → cC | d

Compute closure(I) where I={[S' → .S, $]}

S' → .S,	$
S → .CC,	$
C → .cC,	c
C → .cC,	d
C → .d,	c
C → .d,	d

Example

Construct sets of LR(1) items for the grammar on previous slide

I_0:	$S' \rightarrow .S,$	$\$$
	$S \rightarrow .CC,$	$\$$
	$C \rightarrow .cC,$	c/d
	$C \rightarrow .d,$	c/d
I_1:	goto(I_0,S)	
	$S' \rightarrow S.,$	$\$$
I_2:	goto(I_0,C)	
	$S \rightarrow C.C,$	$\$$
	$C \rightarrow .cC,$	$\$$
	$C \rightarrow .d,$	$\$$
I_3:	goto(I_0,c)	
	$C \rightarrow c.C,$	c/d
	$C \rightarrow .cC,$	c/d
	$C \rightarrow .d,$	c/d
I_4:	goto(I_0,d)	
	$C \rightarrow d.,$	c/d
I_5:	goto(I_2,C)	
	$S \rightarrow CC.,$	$\$$
I_6:	goto(I_2,c)	
	$C \rightarrow c.C,$	$\$$
	$C \rightarrow .cC,$	$\$$
	$C \rightarrow .d,$	$\$$
I_7:	goto(I_2,d)	
	$C \rightarrow d.,$	$\$$
I_8:	goto(I_3,C)	
	$C \rightarrow cC.,$	c/d
I_9:	goto(I_6,C)	
	$C \rightarrow cC.,$	$\$$

Construction of Canonical LR parse table

- Construct $C=\{I_{0, \ldots,} I_n\}$ the sets of LR(1) items.

- If $[A \rightarrow \alpha.a\beta, b]$ is in I_i and $goto(I_i, a)=I_j$
  then action[i,a]=shift j

- If $[A \rightarrow \alpha., a]$ is in I_i
  then action[i,a] reduce $A \rightarrow \alpha$

- If $[S' \rightarrow S., \$]$ is in I_i
  then action[i,\$] = accept

- If $goto(I_i, A) = I_j$ then goto[i,A] = j for all non terminals A

Parse table

State	c	d	$	S	C
0	s3	s4		1	2
1			acc		
2	s6	s7			5
3	s3	s4			8
4	r3	r3			
5			r1		
6	s6	s7			9
7			r3		
8	r2	r2			
9			r2		

Notes on Canonical LR Parser

- Consider the grammar discussed in the previous two slides. The language specified by the grammar is c*dc*d.

- When reading input cc…dcc…d the parser shifts cs into stack and then goes into state 4 after reading d. It then calls for reduction by C→d if following symbol is c or d.

- IF $ follows the first d then input string is c*d which is not in the language; parser declares an error

- On an error canonical LR parser never makes a wrong shift/reduce move. It immediately declares an error

- Problem: Canonical LR parse table has a large number of states

LALR Parse table

- Look Ahead LR parsers

- Consider a pair of similar looking states (same kernel and different lookaheads) in the set of LR(1) items
  I_4: C → d. , c/d I_7: C → d., $

- Replace I_4 and I_7 by a new state I_{47} consisting of (C → d., c/d/$)

- Similarly I_3 & I_6 and I_8 & I_9 form pairs

- Merge LR(1) items having the same core

Construct LALR parse table

- Construct C={I_0,……,I_n} set of LR(1) items
- For each core present in LR(1) items find all sets having the same core and replace these sets by their union
- Let C' = {J_0,…….,J_m} be the resulting set of items
- Construct action table as was done earlier
- Let J = I_1 U I_2…….U I_k

  since I_1 , I_2……., I_k have same core, goto(J,X) will have he same core

  Let K=goto(I_1,X) U goto(I_2,X)……goto(I_k,X) the goto(J,X)=K

LALR parse table …

State	c	d	$	S	C
0	s36	s47		1	2
1			acc		
2	s36	s47			5
36	s36	s47			89
47	r3	r3	r3		
5			r1		
89	r2	r2	r2		

Notes on LALR parse table

- Modified parser behaves as original except that it will reduce C→d on inputs like ccd. The error will eventually be caught before any more symbols are shifted.

- In general core is a set of LR(0) items and LR(1) grammar may produce more than one set of items with the same core.

- Merging items never produces shift/reduce conflicts but may produce reduce/reduce conflicts.

- SLR and LALR parse tables have same number of states.

Notes on LALR parse table…

- Merging items may result into conflicts in LALR parsers which did not exist in LR parsers

- New conflicts can not be of shift reduce kind:
  - Assume there is a shift reduce conflict in some state of LALR parser with items
    $\{[X \rightarrow \alpha., a], [Y \rightarrow \gamma.a\beta, b]\}$
  - Then there must have been a state in the LR parser with the same core
  - Contradiction; because LR parser did not have conflicts

- LALR parser can have new reduce-reduce conflicts
  - Assume states
    $\{[X \rightarrow \alpha., a], [Y \rightarrow \beta., b]\}$ and $\{[X \rightarrow \alpha., b], [Y \rightarrow \beta., a]\}$
  - Merging the two states produces
    $\{[X \rightarrow \alpha., a/b], [Y \rightarrow \beta., a/b]\}$

Notes on LALR parse table…

- LALR parsers are not built by first making canonical LR parse tables
- There are direct, complicated but efficient algorithms to develop LALR parsers
- Relative power of various classes
  - SLR(1) ≤ LALR(1) ≤ LR(1)
  - SLR(k) ≤ LALR(k) ≤ LR(k)
  - LL(k) ≤ LR(k)

Error Recovery

- An error is detected when an entry in the action table is found to be empty.
- Panic mode error recovery can be implemented as follows:
  - scan down the stack until a state S with a goto on a particular nonterminal A is found.
  - discard zero or more input symbols until a symbol a is found that can legitimately follow A.
  - stack the state goto[S,A] and resume parsing.
- **Choice of A:** Normally these are non terminals representing major program pieces such as an expression, statement or a block. For example if A is the nonterminal stmt, a might be semicolon or end.

Parser Generator

- Some common parser generators
  - YACC: **Y**et **A**nother **C**ompiler **C**ompiler
  - Bison: GNU Software
  - ANTLR: **AN**other **T**ool for **L**anguage **R**ecognition
- Yacc/Bison source program specification (accept LALR grammars)

  declaration

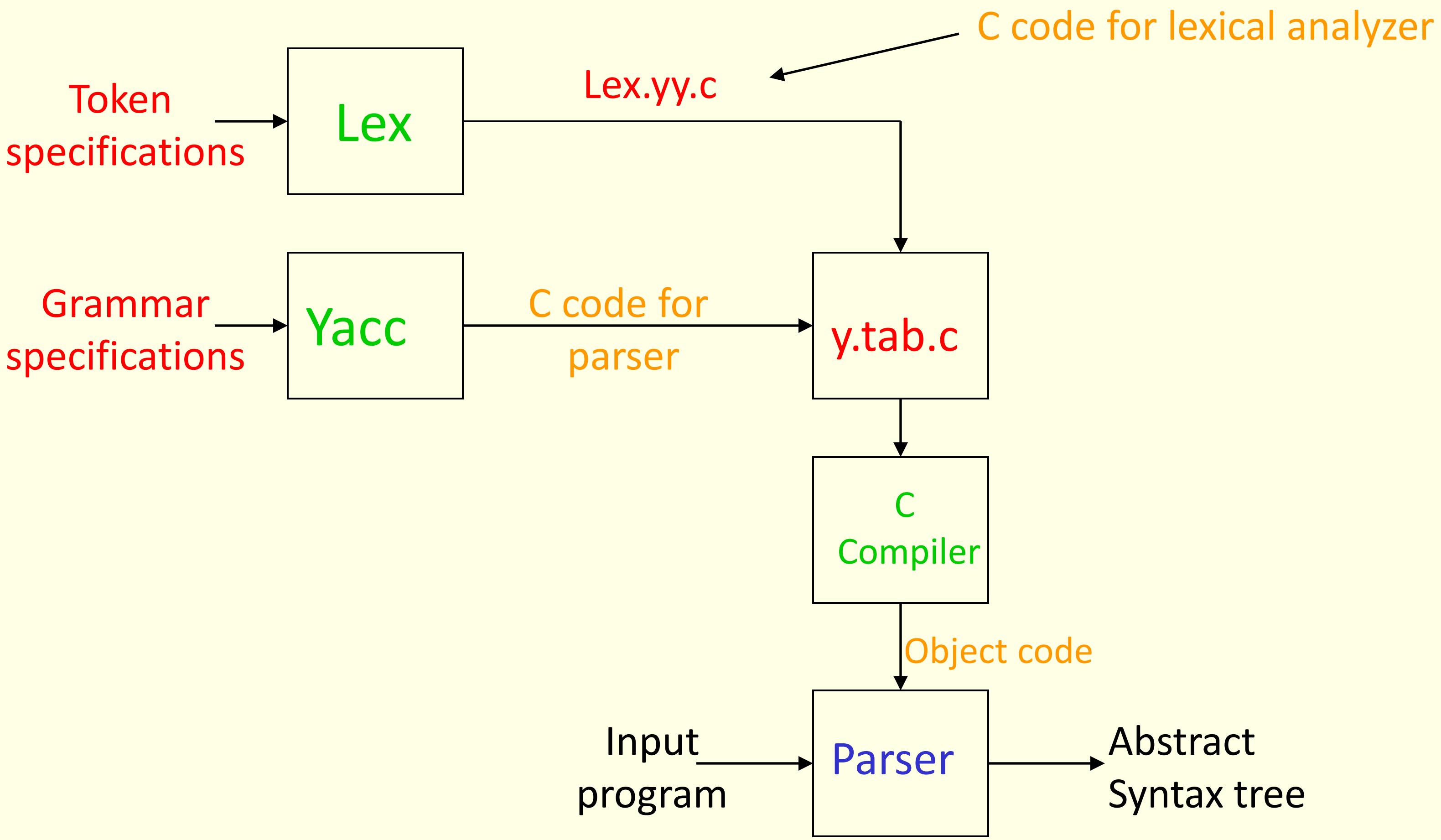
  %%

  translation rules

  %%

  supporting C routines

Yacc and Lex schema



Refer to YACC Manual

Bottom up parsing …

- A more powerful parsing technique
- LR grammars – more expensive than LL
- Can handle left recursive grammars
- Can handle virtually all the programming languages
- Natural expression of programming language syntax
- Automatic generation of parsers (Yacc, Bison etc.)
- Detects errors as soon as possible
- Allows better error recovery